

2  
SECTION A  
[48 marks]

*Answer all the questions in this section. All questions carry equal marks.*

1. A binary operation  $\Delta$  is defined on the set of real numbers,  $R$ , by  $x \Delta y = x + y + 10$ . Find the:
- (a) identity element;
- (b) inverses of 3 and  $-5$  under  $\Delta$ .

2. Evaluate  $\int_2^4 \left( \frac{x^3 + 3}{x^2} \right) dx$ .

3. (a) Two functions  $f$  and  $g$  are defined on the set of real numbers,  $R$ , by  $f: x \rightarrow x^2 - 1$  and  $g: x \rightarrow x + 2$ . Find  $f \circ g(-2)$ .
- (b) A bus has 6 seats and there are 8 passengers. In how many ways can the bus be filled?

4. Express  $\frac{1}{x^2 - 16}$  in partial fractions.

5. The table shows the marks scored by some students in a class test.

|                 |         |         |         |         |         |         |         |
|-----------------|---------|---------|---------|---------|---------|---------|---------|
| Marks           | 11 - 14 | 15 - 18 | 19 - 22 | 23 - 26 | 27 - 30 | 31 - 34 | 35 - 38 |
| No. of Students | 4       | 5       | 18      | 31      | 25      | 14      | 3       |

- (a) Draw a histogram for the distribution.
- (b) Use the histogram to estimate the modal score, correct to **one** decimal place.
6. A bag contains 10 black and 5 yellow identical balls. **Two** balls are picked at random from the bag one after the other **without** replacement. Calculate the probability that the are:
- (a) **both** black;
- (b) of the **same** colour.
7. Forces  $F_1(24 \text{ N}, 120^\circ)$ ,  $F_2(18 \text{ N}, 240^\circ)$  and  $F_3(12 \text{ N}, 300^\circ)$  act at a point. Find, correct to **two** decimal places, the magnitude of their resultant force.
8. The vectors  $\mathbf{p}$ ,  $\mathbf{q}$  and  $\mathbf{r}$  are mutually perpendicular with  $|\mathbf{q}| = 3$  and  $|\mathbf{r}| = \sqrt{5.4}$ . If the vectors  $\mathbf{X} = 3\mathbf{p} + 5\mathbf{q} + 7\mathbf{r}$  and  $\mathbf{Y} = 2\mathbf{p} + 3\mathbf{q} - 5\mathbf{r}$  are perpendicular, find  $|\mathbf{p}|$ .

Answer **four** questions **only** from this section with **at least one** question from **each** part

All questions carry equal marks.

PART I  
PURE MATHEMATICS

9. (a) If  $(p + 1)x^2 + 4px + (2p + 3) = 0$  has equal roots, find the integral value of  $p$ .
- (b) Solve for  $x$  and  $y$  in the equations:  $\log(x - 1) + 2 \log y = 2 \log 3$ ;  
 $\log x + \log y = \log 6$ .
10. (a) Differentiate  $y = \frac{3x}{1 + x^2}$  with respect to  $x$ .
- (b) Find the equation of the circle that passes through  $(2, 3)$ ,  $(4, 2)$  and  $(1, 11)$ .
11. When the terms of a Geometric Progression (G.P.) with common ratio  $r = 2$  is added to the corresponding terms of an Arithmetic Progression (A.P.), a new sequence is formed. If the first terms of the G.P. and A.P. are the same and the first three terms of the new sequence are 3, 7 and 11 respectively, find the  $n^{\text{th}}$  term of the new sequence.

PART II

STATISTICS AND PROBABILITY

12. (a) The probabilities that Golu, Kofi and Barry will win a competition are  $\frac{1}{3}$ ,  $\frac{2}{5}$  and  $\frac{1}{2}$  respectively. Find the probability that **only two** of them wins the competition.
- (b) Ten eggs are picked successively **with** replacement from a lot containing 10% defective eggs. Find the probability that **at least two** are defective.
13. The marks awarded by three examiners are given in the table:

| Candidate    | A  | B  | C  | D  | E  | F  | G  | H  | I  | J  |
|--------------|----|----|----|----|----|----|----|----|----|----|
| Examiner I   | 90 | 88 | 71 | 65 | 32 | 72 | 70 | 41 | 38 | 14 |
| Examiner II  | 89 | 92 | 70 | 68 | 35 | 66 | 72 | 39 | 40 | 16 |
| Examiner III | 88 | 89 | 71 | 67 | 36 | 70 | 69 | 38 | 39 | 15 |

- (a) Calculate the Spearman's rank correlation coefficient of the marks awarded by:
- Examiners I and II;
  - Examiners I and III;
  - Examiners II and III.
- (b) Use your results in (a) to determine which of the examiners agree most.

## VECTORS AND MECHANICS

14. The ends **X** and **Y** of an inextensible string 27 m long are fixed at two points on the same horizontal line which are 20 m apart. A particle of mass 7.5 kg is suspended from a point **P** on the string 12 m from **X**.
- (a) Illustrate this information in a diagram.
- (b) Calculate, correct to **two** decimal places,  $\angle YXP$  and  $\angle XYP$ .
- (c) Find, correct to the **nearest** hundredth, the magnitudes of the tensions in the string.  
[Take  $g = 10 \text{ m s}^{-2}$ ]
15. A particle **P** moves in a plane such that at time  $t$  seconds, its velocity,  $\mathbf{v} = (2t\mathbf{i} - t^3\mathbf{j}) \text{ m s}^{-1}$ .
- (a) Find, when  $t = 2$ , the magnitude of the:
- (i) velocity of **P**.
- (ii) acceleration of **P**.
- (b) Given that **P** is at the point with position vector  $(3\mathbf{i} + 2\mathbf{j})$  when  $t = 1$ , find the position vector of **P** when  $t = 2$ .

**END OF PAPER**

$$1) 2t^2 + 10 + 7t + 3 = 0$$

$$1) 2 = 10 + 7t + 3$$

$$u = 0$$

$$1) (2t^2 + 10 + 7t + 3) = 2t^2 + 7t + 13$$

$$x = y$$

$$3p + 5q + 7r = 2p + 3q - 5r$$